**MIDTERM REVIEW**

**Propositional Logic**

Simplify the following formulas and provide valid reasoning using laws of logic:

Biconditional identity

Conditional statements x2

de Morgan’s Law

de Morgan’s Law

Distribution

Negation

Identity

Associativity

Negation

Domination

**Predicate Logic**

1. Let be “x is an even number” and . What are the truth values of ?

a. Since says there exists an x that is an even number, and , then this evaluates to true since 2 is a value of x that satisfies the condition.

b. Since says that all of x is an even number, and , then this evaluates to false since 3 is a value of x that does not satisfy the condition.

1. Your COT3100 professor wants to grade every exam for at least one of the 15 sections that they teach. What is one way to represent this statement as a predicate?

* Let x represent a single exam in the set of all exams, and y represent a section in the set of all sections.
* Now P(x): x is graded; Q(x,y): x is an exam in section y
* So now to represent that x is graded and is an exam in section y, we can say for now that .
* Filling in the quantifiers gets

**Rules of Inference**

1. Accepting these premises as true:

Conclude

1. premise
2. existential instantiation on (1)
3. simplification on (2)
4. same as (3)
5. premise
6. universal instantiation on (5)
7. MP on (3) and (6)
8. conjunction on (4) and (7)
9. existential generalization on (8)

## Proofs

Prove using contradiction:

1. Prove that for is an integer, if is odd, then is odd.

Let Now let’s assume n is not odd, so even.

If n is even, then , for some integer k. Substituting n into the first half gets:

Since is even when n is even. This is a contradiction, meaning that the original expression must be odd when n is odd.

Prove using contraposition:

1. Prove that for all integers and , if is odd, then

First lets look at this statement as a compound proposition :

is odd

Taking the contrapositive gives

is even

So, if , then is even

Due to the definition of even numbers, and since we are assuming that , as well. When they are the same then the sum would be , which satisfies the definition of even numbers.

Therefore, by definition of contrapositive, if is odd, then .

Prove using cases:

1. Prove that if is an integer, then is an even integer.

Since there are only two types of integers, odd and even, this proof is split into two cases:

Case 1:

If , then

By definition of even numbers, this holds true.

Case 2:

If , then:

which is also an even number.

Therefore, by proof of cases, for all integers , is an even number.

**Sets**

1. Prove using laws of sets that .

relative complement

distributive law

null law

identity law

1. Prove that if .

Accepting the premises:

Consider

by definition of union on (1)

because and and by definition of subset

Since no matter if it originated from A or B, then by universal generalization and the definition of subset .

**Functions**

1. Let , and . Find all values of .

**Number Theory:**

Division

1. Show if then

Accepting the premises, for some integers t and s:

Then

Thus .

1. Prove that if 5 divides n with remainder 3, 5 divides with remainder 2.

If 5 divides n with remainder 3, and we have . Then:

Which is divisible by 5 with a remainder of 2

Congruency

1. Are they congruent?

and , therefore they are not congruent

QUESTIONS TO SOLVE FOR PRACTICE (RECOMMENDED FROM TEXTBOOK)

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Please do not limit yourselves with these questions. Try to solve as many problems as you can.

Best of luck,

Mesut